Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|

Centre Number | Candidate Number

# Pearson Edexcel Level 3 GCE

## Monday 24 June 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **9FM0/4A**

# Further Mathematics

## Advanced

## PAPER 4A: Further Pure Mathematics 2

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  *– there may be more space than you need.*
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  *– use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

**1.** **In this question you must show detailed reasoning.**

Use Fermat's Little Theorem to determine the least positive residue of

$$21^{80} \pmod{23}$$

**(4)**

**Question 1 continued**

**(Total for Question 1 is 4 marks)**

**2.** Determine a closed form for the recurrence system

$$u_1 = 4 \qquad u_2 = 6$$
$$u_{n+2} = 6u_{n+1} - 9u_n \qquad (n = 1, 2, 3, \ldots)$$

**(5)**

**Question 2 continued**

**Question 2 continued**

**Question 2 continued**

**(Total for Question 2 is 5 marks)**

**3.** **In this question you must show all stages of your working.**
**Solutions relying on calculator technology are not acceptable.**

(a) Use the Euclidean Algorithm to determine the highest common factor $h$ of 234 and 96

**(3)**

(b) Hence determine integers $a$ and $b$ such that

$$234a + 96b = h$$

**(3)**

(c) Solve the congruence equation

$$96x \equiv 36 \pmod{234}$$

**(5)**

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**(Total for Question 3 is 11 marks)**

**4.**

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 0 \\ 2 & p & -2 \\ 0 & -2 & 2 \end{pmatrix} \quad \text{where } p \text{ is a constant}$$

Given that $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ is an eigenvector of **A**,

(a) determine the eigenvalue corresponding to this eigenvector.

**(2)**

(b) Hence show that $p = 3$

**(1)**

(c) Determine

(i) the remaining eigenvalues of **A**,

(ii) corresponding eigenvectors for these eigenvalues.

**(6)**

(d) Hence determine a matrix **P** and a diagonal matrix **D** such that $\mathbf{A} = \mathbf{PDP}^{\mathrm{T}}$

**(3)**

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**(Total for Question 4 is 12 marks)**

**5.** (i) A circle $C$ in the complex plane is defined by the locus of points satisfying

$$|z - 3i| = 2|z|$$

(a) Determine a Cartesian equation for $C$, giving your answer in simplest form.

**(3)**

(b) On an Argand diagram, shade the region defined by

$$\{z \in \mathbb{C} : |z - 3i| > 2|z|\}$$

**(2)**

(ii) The transformation $T$ from the $z$-plane to the $w$-plane is given by

$$w = z^3$$

(a) Describe the geometric effect of $T$.

**(2)**

The region $R$ in the $z$-plane is given by

$$\left\{z \in \mathbb{C} : 0 < \arg z < \frac{\pi}{4}\right\}$$

(b) On a **different** Argand diagram, sketch the image of $R$ under $T$.

**(2)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 9 marks)**

**6.** **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

$$I_n = \int \frac{\cos(nx)}{\sin x}\,\mathrm{d}x \qquad n \geqslant 1$$

(a) Show that, for $n \geqslant 1$

$$I_{n+2} = \frac{2\cos(n+1)x}{n+1} + I_n$$

**(6)**

(b) Hence determine the exact value of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos(5x)}{\sin x}\,\mathrm{d}x$$

giving the answer in the form $a + b\ln c$ where $a$, $b$ and $c$ are rational numbers to be found.

**(5)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 11 marks)**

**7.** The set of matrices $G = \{\mathbf{I}, \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}\}$ where

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

with the operation $\otimes_2$ of matrix multiplication with entries evaluated modulo 2, forms a group.

(a) Show that **B** is an element of order 3 in $G$.

**(2)**

(b) Determine the orders of the other elements of $G$.

**(3)**

(c) Give a reason why $G$ is **not** isomorphic to

(i) a cyclic group of order 6

(ii) the group of symmetries of a regular hexagon.

**(2)**

The group $H$ of permutations of the numbers 1, 2 and 3 contains the following elements, denoted in two-line notation,

$$e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad a = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$c = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \quad d = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \quad f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

(d) Determine an isomorphism between the groups $G$ and $H$.

**(3)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 10 marks)**

**8.**


**Figure 1**

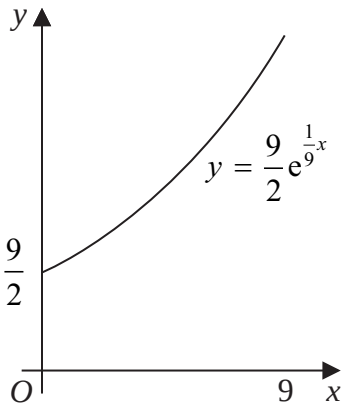


**Figure 2**

Figure 1 shows a French horn with a detachable bell section.

The shape of the bell section can be modelled by rotating an exponential curve through 360° about the $x$-axis, where units are centimetres.

The model uses the curve shown in Figure 2, with equation

$$y = \frac{9}{2}e^{\frac{1}{9}x} \qquad 0 \leqslant x \leqslant 9$$

(a) Show that, according to this model, the external surface area of the bell section is given by

$$K\int_0^9 e^{\frac{1}{9}x}\sqrt{4 + e^{\frac{2}{9}x}}\,dx$$

where $K$ is a real constant to be determined.

**(3)**

(b) Use the substitution $u = e^{\frac{1}{9}x}$ to show that

$$\int_0^9 e^{\frac{1}{9}x}\sqrt{4 + e^{\frac{2}{9}x}}\,dx = 9\int_a^b \frac{2u + u^3}{\sqrt{4u^2 + u^4}}du + 18\int_a^b \frac{1}{\sqrt{4 + u^2}}du$$

where $a$ and $b$ are constants to be determined.

**(5)**

Hence, using algebraic integration,

(c) determine, according to the model, the external surface area of the bell section of the horn, giving your answer to 3 significant figures.

**(5)**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 13 marks)**

**TOTAL FOR PAPER IS 75 MARKS**